UNIT- 2 Content

Output Primitives

1. Point and Lines
2. Line Drawing Algorithms – DDA, Bresenhams Line algorithm
3. Circle Generation Algorithm
4. Ellipse Generating Algorithm

2.1 Points

Point plotting is accomplished by converting a single coordinate position furnished by an application program into appropriate operations for the output device in use.

With a CRT monitor, for example, the electron beam is turned on to illuminate the screen phosphor at the selected location. How the electron beam is positioned depends on the display technology.

A random-scan (vector) system stores point-plotting instructions in the display list, and coordinate values in these instructions are converted to deflection voltages that position the electron beam at the screen locations to be plotted during each refresh cycle.

For a black and-white raster system, on the other hand, a point is plotted by setting the bit value corresponding to A specified screen position within the frame buffer to 1.

To load a specified color into the frame buffer at a position corresponding to column x along scan line y, we will assume we have available a low-level procedure of the form Figure 3-1

```
setPixel (x, y)
```

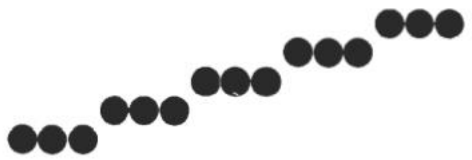


Figure 3-1
Stairstep effect (jaggies) produced when a line is generated as a series of pixel positions.

We sometimes will also want to be able to retrieve the current framebuffer intensity setting for a specified location. We accomplish this with the low-level function

```
getpixel (x, y)
```

2.2 Line Drawing Algorithms

The Cartesian slope-intercept equation for a straight line is

$$y = m . x + c \qquad (3.1)$$

here m representing the slope of the line and b is the intercept.

Given that the two endpoints of a line segment are (x1, y1) and (x2, y2) in Fig. 3-3

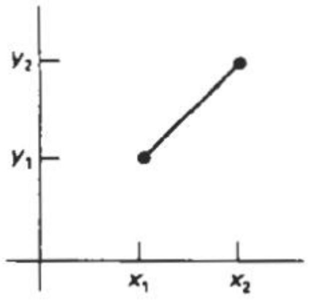


Figure 3-3
Line path between endpoint positions (x_1, y_1) and (x_2, y_2).

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad (3\text{-}2)$$

$$b = y_1 - m \cdot x_1 \qquad (3\text{-}3)$$

Therefore,

$$\Delta y = m\,\Delta x \qquad (3\text{-}4)$$

$$\Delta x = \frac{\Delta y}{m} \qquad (3\text{-}5)$$

1. For lines with slope magnitudes **| m | < 1**, Δx can be set proportional to a small horizontal deflection voltage and the corresponding vertical deflection is then set proportional to Δy as calculated from Eq. 3-4.
2. For lines with slope magnitudes **| m | > 1**, Δy can be set proportional to a small vertical deflection voltage and the corresponding horizontal deflection then set proportional to Δx as calculated from Eq. 3-5.
3. For m = 1, Δx = Δy and the horizontal and vertical deflections voltages are equal. In each case, a smooth line with slope m is generated between the specified endpoints

2.2.1 Digital Differential Analyzer (DDA) Algorithm

The digital differential analyzer (DDA) is a scan-conversion line algorithm based on fiigure 3-4 calculating either Δy or Δx, using Eq. 3-4 or Eq. 3-5.

1. Consider first a line with positive slope, as shown in Fig. 3-3.
   If the slope is less than or equal to 1, we sample at unit x intervals ($\Delta x = 1$) and compute each successive y value as

$$y_{k+1} = y_k + m \tag{3-6}$$

2. For lines with a positive slope greater than 1, we reverse the roles of x and y.
   That is, we sample at unit y intervals ($\Delta y = 1$) and calculate each succeeding x value as

$$x_{k+1} = x_k + \frac{1}{m} \tag{3-7}$$

3. Equations 3-6 and 3-7 are based on the assumption that lines are to be processed from the left endpoint to the right endpoint (Fig. 3-3). If this processing is reversed, so that the starting endpoint is at the right, then either we have $\Delta x = -1$ and

$$y_{k+1} = y_k - m \tag{3-8}$$

or (when the slope is greater than 1) we have $\Delta y = -1$ with

$$x_{k+1} = x_k - \frac{1}{m} \tag{3-9}$$

<u>DDA Algorithm</u>

```
#define ROUND(a) ((int)(a+0.5))

void lineDDA (int xa, int ya, int xb, int yb)
{
  int dx = xb - xa, dy = yb - ya, steps, k;
  float xIncrement, yIncrement, x = xa, y = ya;

  if (abs (dx) > abs (dy)) steps = abs (dx);
  else steps = abs  dy);
  xIncrement = dx / (float) steps;
  yIncrement = dy / (float) steps;
```

```
    setPixel (ROUND(x), ROUND(y));
    for (k=0; k<steps; k++) {
      x += xIncrement;
      y += yIncrement;
      setPixel (ROUND(x), ROUND(y));
    }
  }
```

Advantages of DDA algorithm

1. The DDA algorithm is a faster method for calculating pixel positions than the direct use of Eq. 3-1.
2. It eliminates the multiplication in Eq. 3-1 by making use of raster characteristics, so that appropriate increments are applied in the x or y direction

Disadvantage of DDA

1. The rounding operations and floating-point arithmetic in procedure lineDDA are still time-consuming

2.2.2 Bresenham's Line Algorithm

An accurate and efficient raster line-generating algorithm, developed by Bresenham, scan converts lines using only incrementa1 integer calculations.

To illustrate Bresenharm's approach, we- first consider the scan-conversion process for lines with positive slope less than 1.

1. Pixel positions along a line path are then determined by sampling at unit x intervals.
2. Starting from the left end point (x0, y0) of a given line, we step to each successive column (x position) and plot the pixel whose scan-line y value is closest to the line path. Figure 3-7 demonstrates the kth step in this process.
3. Assuming we have determined that the pixel at (x_k, y_k) is to be displayed, we next need to decide which pixel to plot in column x_{k+1} . Therefore, Our choices are the pixels at positions (x_{k+1} , y_k) and (x_{k+1} , y_{k+1})

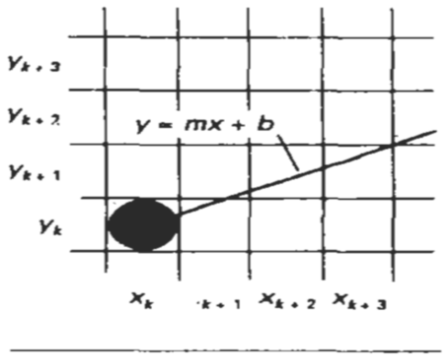


Figure 3-7

4. At sampling position x_{k+l}, we label vertical pixel separations from the mathematical line path as d, and d2 (Fig. 3-8).

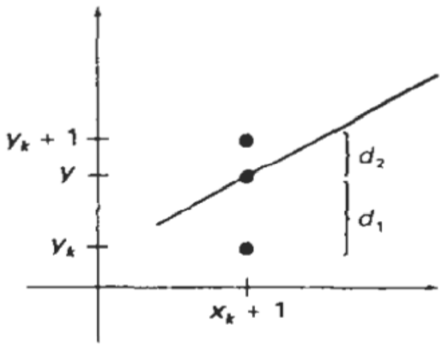


Figure 3-8

$$y = m(x_k + 1) + b \tag{3-10}$$

Then

$$\begin{aligned} d_1 &= y - y_k \\ &= m(x_k + 1) + b - y_k \end{aligned}$$

and

$$\begin{aligned} d_2 &= (y_k + 1) - y \\ &= y_k + 1 - m(x_k + 1) - b \end{aligned}$$

The difference between these two separations is

$$d_1 - d_2 = 2m(x_k + 1) - 2y_k + 2b - 1 \tag{3-11}$$

$$\begin{aligned} p_k &= \Delta x(d_1 - d_2) \\ &= 2\Delta y \cdot x_k - 2\Delta x \cdot y_k + c \end{aligned} \tag{3-12}$$

$$p_{k+1} = 2\Delta y \cdot x_{k+1} - 2\Delta x \cdot y_{k+1} + c$$

Subtracting Eq. 3-12 from the preceding equation, we have

$$p_{k+1} - p_k = 2\Delta y(x_{k+1} - x_k) - 2\,\Delta x(y_{k+1} - y_k)$$

But $x_{k+1} = x_k + 1$, so that

$$p_{k+1} = p_k + 2\Delta y - 2\Delta x(y_{k+1} - y_k) \tag{3-13}$$

where the term $y_{k+1} - y_k$ is either 0 or 1, depending on the sign of parameter p_k.

This recursive calculation of decision parameters is performed at each integer x position, starting at the left coordinate endpoint of the line. The first parameter, p_0, is evaluated from Eq. 3-12 at the starting pixel position (x_0, y_0) and with m evaluated as $\Delta y/\Delta x$:

$$p_0 = 2\Delta y - \Delta x \tag{3-14}$$

Bresenham's Line-Drawing Algorithm for $|m| < 1$

1. Input the two line endpoints and store the left endpoint in (x_0, y_0).
2. Load (x_0, y_0) into the frame buffer; that is, plot the first point.
3. Calculate constants Δx, Δy, $2\Delta y$, and $2\Delta y - 2\Delta x$, and obtain the starting value for the decision parameter as

$$p_0 = 2\Delta y - \Delta x$$

4. At each x_k along the line, starting at $k = 0$, perform the following test: If $p_k < 0$, the next point to plot is $(x_k + 1, y_k)$ and

$$p_{k+1} = p_k + 2\Delta y$$

Otherwise, the next point to plot is $(x_k + 1, y_k + 1)$ and

$$p_{k+1} = p_k + 2\Delta y - 2\Delta x$$

5. Repeat step 4 Δx times.

Example 3-1 Bresenham Line Drawing

To illustrate the algorithm, we digitize the line with endpoints (20, 10) and (30, 18). This line has a slope of 0.8, with

$$\Delta x = 10, \qquad \Delta y = 8$$

The initial decision parameter has the value

$$\begin{aligned} p_0 &= 2\Delta y - \Delta x \\ &= 6 \end{aligned}$$

and the increments for calculating successive decision parameters are

$$2\Delta y = 16, \qquad 2\Delta y - 2\Delta x = -4$$

We plot the initial point $(x_0, y_0) = (20, 10)$, and determine successive pixel positions along the line path from the decision parameter as

k	p_k	(x_{k+1}, y_{k+1})	k	p_k	(x_{k+1}, y_{k+1})
0	6	(21, 11)	5	6	(26, 15)
1	2	(22, 12)	6	2	(27, 16)
2	−2	(23, 12)	7	−2	(28, 16)
3	14	(24, 13)	8	14	(29, 17)
4	10	(25, 14)	9	10	(30, 18)

A plot of the pixels generated along this line path is shown in Fig. 3-9.

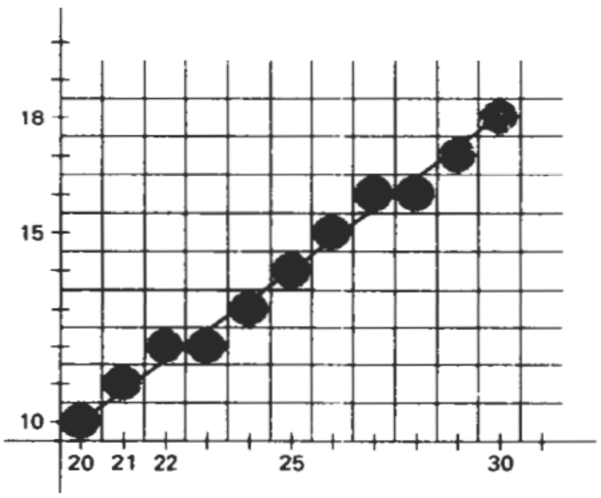


Figure 3-9
Pixel positions along the line path between endpoints (20, 10) and (30, 18), plotted with Bresenham's line algorithm.

2.3 CIRCLE-GENERATING ALGORITHM

Properties of Circles A circle is defined as the set of points that are all at a given distance r from a center position (x,, y,) (Fig. 3-12). This distance relationship is expressed by the Pythagorean theorem in Cartesian coordinates as

$$(x - x_c)^2 + (y - y_c)^2 = r^2$$

$$y = y_c \pm \sqrt{r^2 - (x_c - x)^2}$$

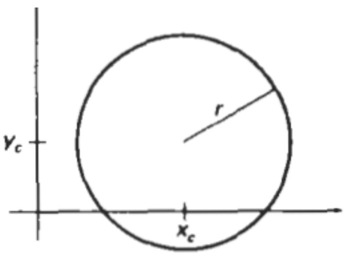


Figure 3-12
Circle with center coordinates (x_c, y_c) and radius r.

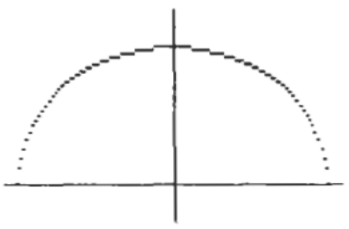


Figure 3-13
Positive half of a circle plotted with Eq. 3-25 and with $(x_c, y_c) = (0, 0)$.

Another way to eliminate the unequal spacing shown in Fig. 3-13 is to calculate points along the circular boundary using polar coordinates r and θ (Fig. 3-12). Expressing the circle equation in parametric polar form yields the pair of equations

$$x = x_c + r\cos\theta$$
$$y = y_c + r\sin\theta \qquad (3\text{-}26)$$

Computation can be reduced by considering the symmetry of circles. The shape of the circle is similar in each quadrant. Circle sections in adjacent octants within one quadrant are symmetric with respect to the 45^o line dividing the two octants. These symmetry conditions are illustrated in Fig.3-14, where a point at position (x, y) on a one-eighth circle sector is mapped into the seven circle points in the other octants of the xy plane.

Midpoint Circle Algorithm

As in the raster line algorithm, we sample at unit intervals and determine the closest pixel position to the specified circle path at each step.

For a given radius r and screen center position (x, y,), we can first set up our algorithm to calculate pixel positions around a circle path centered at the coordinate origin (0,0). Then each calculated position (x, y) is moved to its proper screen position by adding x, to x and y, toy.

Along the circle section from x = 0 to x = y in the first quadrant, the slope of the curve varies from 0 to -1.

Therefore, we can take unit steps in the positive x direction over this octant and use a decision parameter to determine which of the two possible y positions is closer to the circle path at each step. Positions in the other seven octants are then obtained by symmetry.

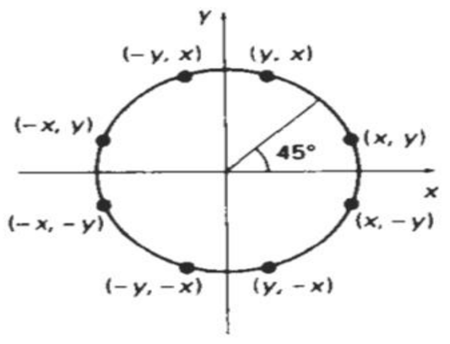


Figure 3-14
Symmetry of a circle. Calculation of a circle point (x, y) in one octant yields the circle points shown for the other seven octants.

To apply the midpoint method, we define a circle function:

$$f_{\text{circle}}(x, y) = x^2 + y^2 - r^2$$

Any point (x, y) on the boundary of the circle with radius r satisfies the equation $f_{\text{circle}}(x, y) = 0$. If the point is in the interior of the circle, the circle function is negative. And if the point is outside the circle, the circle function is positive. To summarize, the relative position of any point (x, v) can be determined by checking the sign of the circle function:

$$f_{\text{circle}}(x, y) \begin{cases} < 0, & \text{if } (x, y) \text{ is inside the circle boundary} \\ = 0, & \text{if } (x, y) \text{ is on the circle boundary} \\ > 0, & \text{if } (x, y) \text{ is outside thé circle boundary} \end{cases} \qquad (3\text{-}28)$$

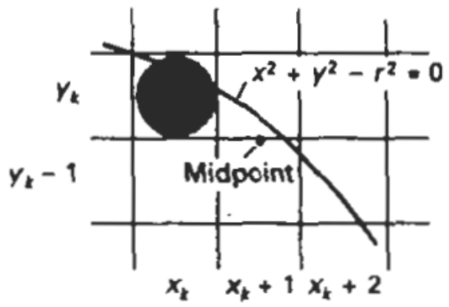


Figure 3-15
Midpoint between candidate pixels at sampling position x_k+1 along a circular path.

Figure 3-15 shows the midpoint between the two candidate pixels at sampling position $x_k + 1$. Assuming we have just plotted the pixel at (x_k, y_k), we next need to determine whether the pixel at position $(x_k + 1, y_k)$ or the one at position $(x_k + 1, y_k - 1)$ is closer to the circle. Our decision parameter is the circle function 3-27 evaluated at the midpoint between these two pixels:

$$\begin{aligned} p_k &= f_{\text{circle}}\left(x_k + 1, y_k - \frac{1}{2}\right) \\ &= (x_k + 1)^2 + \left(y_k - \frac{1}{2}\right)^2 - r^2 \end{aligned} \tag{3-29}$$

If $p_k < 0$, this midpoint is inside the circle and the pixel on scan line y_k is closer to the circle boundary. Otherwise, the midposition is outside or on the circle boundary, and we select the pixel on scanline $y_k - 1$.

Successive decision parameters are obtained using incremental calculations. We obtain a recursive expression for the next decision parameter by evaluating the circle function at sampling position $x_{k+1} + 1 = x_k + 2$:

$$\begin{aligned} p_{k+1} &= f_{\text{circle}}\left(x_{k+1} + 1, y_{k+1} - \frac{1}{2}\right) \\ &= [(x_k + 1) + 1]^2 + \left(y_{k+1} - \frac{1}{2}\right)^2 - r^2 \end{aligned}$$

or

$$p_{k+1} = p_k + 2(x_k + 1) + (y_{k+1}^2 - y_k^2) - (y_{k+1} - y_k) + 1 \qquad (3\text{-}30)$$

where y_{k+1} is either y_k or y_{k-1}, depending on the sign of p_k.

Increments for obtaining p_{k+1} are either $2x_{k+1} + 1$ (if p_k is negative) or $2x_{k+1} + 1 - 2y_{k+1}$. Evaluation of the terms $2x_{k+1}$ and $2y_{k+1}$ can also be done incrementally as

$$2x_{k+1} = 2x_k + 2$$
$$2y_{k+1} = 2y_k - 2$$

At the start position $(0, r)$, these two terms have the values 0 and $2r$, respectively. Each successive value is obtained by adding 2 to the previous value of $2x$ and subtracting 2 from the previous value of $2y$.

The initial decision parameter is obtained by evaluating the circle function at the start position $(x_0, y_0) = (0, r)$:

$$p_0 = f_{\text{circle}}\left(1, r - \frac{1}{2}\right)$$
$$= 1 + \left(r - \frac{1}{2}\right)^2 - r^2$$

or

$$p_0 = \frac{5}{4} - r$$

If the radius r is specified as an integer, we can simply round p_0 to

$$p_0 = 1 - r \qquad \text{(for } r \text{ an integer)}$$

since all increments are integers.

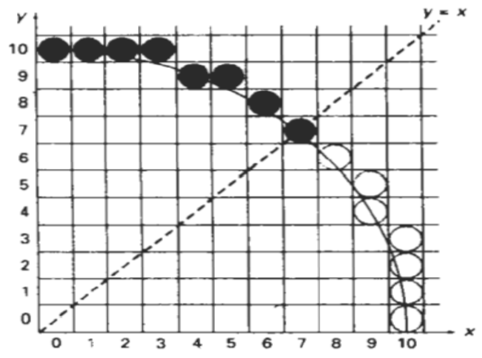


Figure 3-16
Selected pixel positions (solid circles) along a circle path with radius $r = 10$ centered on the origin, using the midpoint circle algorithm. Open circles show the symmetry positions in the first quadrant.

Example 3-2 Midpoint Circle-Drawing

Given a circle radius $r = 10$, we demonstrate the midpoint circle algorithm by determining positions along the circle octant in the first quadrant from $x = 0$ to $x = y$. The initial value of the decision parameter is

$$p_0 = 1 - r = -9$$

For the circle centered on the coordinate origin, the initial point is $(x_0, y_0) = (0, 10)$, and initial increment terms for calculating the decision parameters are

$$2x_0 = 0, \qquad 2y_0 = 20$$

Successive decision parameter values and positions along the circle path are calculated using the midpoint method as

k	p_k	(x_{k+1}, y_{k+1})	$2x_{k+1}$	$2y_{k+1}$
0	−9	(1, 10)	2	20
1	−6	(2, 10)	4	20
2	−1	(3, 10)	6	20
3	6	(4, 9)	8	18
4	−3	(5, 9)	10	18
5	8	(6, 8)	12	16
6	5	(7, 7)	14	14

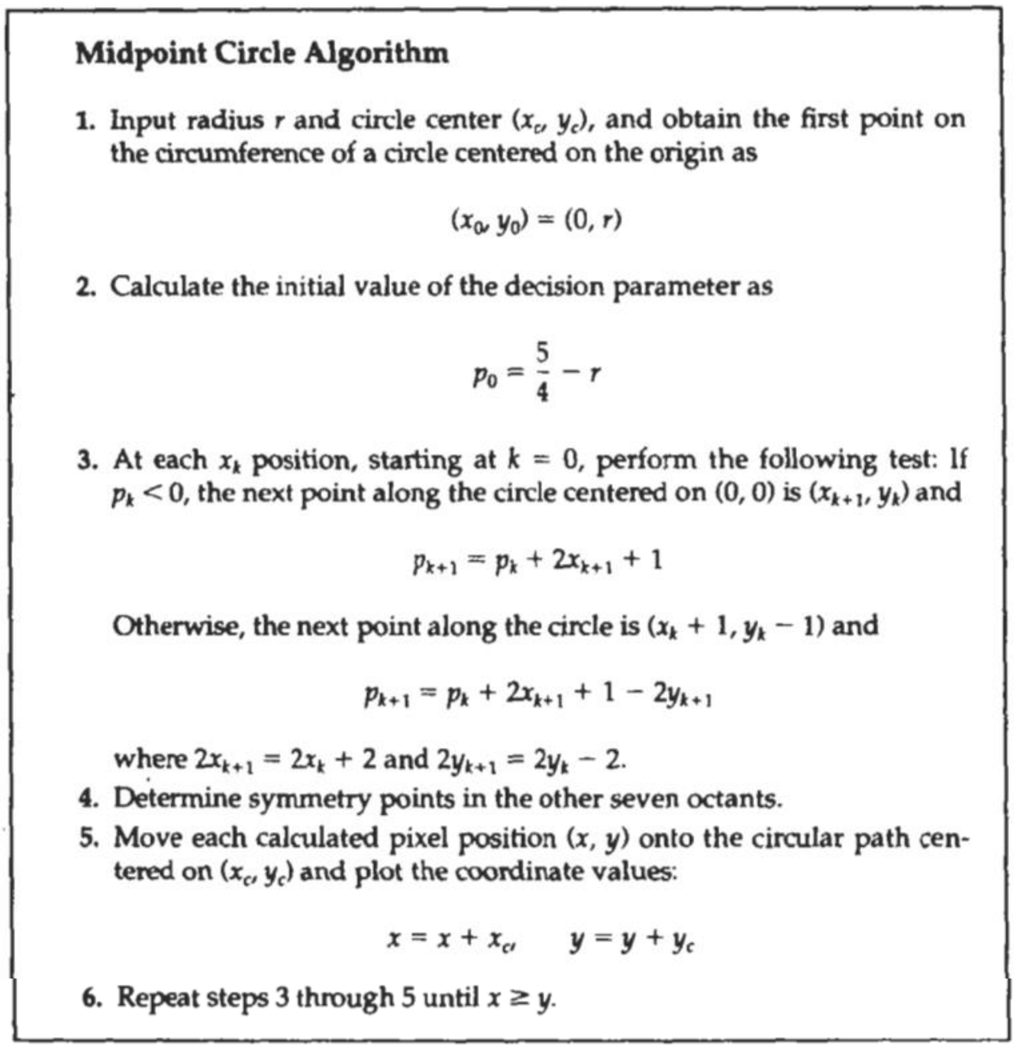
A plot of the generated pixel positions in the first quadrant is shown in Fig. 3-10.

```
void circleMidpoint (int xCenter, int yCenter, int radius)
{
  int x = 0;
  int y = radius;
  int p = 1 - radius;
  void circlePlotPoints (int, int, int, int);

  /* Plot first set of points */
  circlePlotPoints (xCenter, yCenter, x, y);

  while (x < y) {
    x++;
    if (p < 0)
      p += 2 * x + 1;
    else {
      y--;
      p += 2 * (x - y) + 1;
    }
    circlePlotPoints (xCenter, yCenter, x, y);
  }
}

void circlePlotPoints (int xCenter, int yCenter, int x, int y]
{
  setPixel (xCenter + x, yCenter + y);
  setPixel (xCenter - x, yCenter + y);
  setPixel (xCenter + x, yCenter - y);
  setPixel (xCenter - x, yCenter - y);
  setPixel (xCenter + y, yCenter + x);
  setPixel (xCenter - y, yCenter + x);
  setPixel (xCenter + y, yCenter - x);
  setPixel (xCenter - y, yCenter - x);
}
```



Midpoint Circle Algorithm

1. Input radius r and circle center (x_c, y_c), and obtain the first point on the circumference of a circle centered on the origin as

$$(x_0, y_0) = (0, r)$$

2. Calculate the initial value of the decision parameter as

$$p_0 = \frac{5}{4} - r$$

3. At each x_k position, starting at $k = 0$, perform the following test: If $p_k < 0$, the next point along the circle centered on (0, 0) is (x_{k+1}, y_k) and

$$p_{k+1} = p_k + 2x_{k+1} + 1$$

Otherwise, the next point along the circle is $(x_k + 1, y_k - 1)$ and

$$p_{k+1} = p_k + 2x_{k+1} + 1 - 2y_{k+1}$$

where $2x_{k+1} = 2x_k + 2$ and $2y_{k+1} = 2y_k - 2$.

4. Determine symmetry points in the other seven octants.
5. Move each calculated pixel position (x, y) onto the circular path centered on (x_c, y_c) and plot the coordinate values:

$$x = x + x_c, \qquad y = y + y_c$$

6. Repeat steps 3 through 5 until $x \geq y$.

2.4 ELLIPSE-GENERATING ALGORITHMS

An ellipse is defined as the set of points such that the sum of the distances from two fixed positions (foci) is the same for all points (Fig. 3-17). If the distances to the two foci from any point P = (x, y) on the ellipse are labeled d1 and d2, then the general equation of an ellipse can be stated as

$$d_1 + d_2 = \text{constant}$$

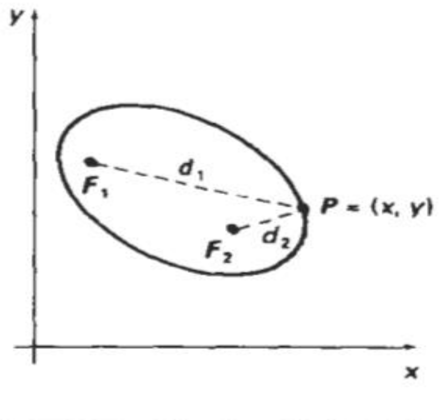


Figure 3-17
Ellipse generated about foci $\mathbf{F}_1$ and $\mathbf{F}_2$.

Expressing distances d_1 and d_2 in terms of the focal coordinates $\mathbf{F}_1 = (x_1, y_1)$ and $\mathbf{F}_2 = (x_2, y_2)$, we have

$$\sqrt{(x - x_1)^2 + (y - y_1)^2} + \sqrt{(x - x_2)^2 + (y - y_2)^2} = \text{constant} \qquad (3\text{-}33)$$

Midpoint Ellipse Algorithm

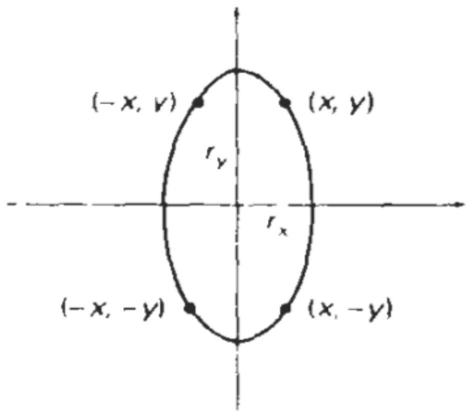


Figure 3-19
Symmetry of an ellipse. Calculation of a point (x, y) in one quadrant yields the ellipse points shown for the other three quadrants.

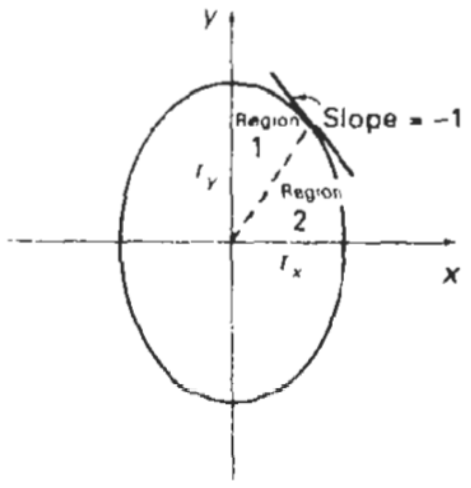


Figure 3-20

$$f_{\text{ellipse}}(x, y) = r_y^2x^2 + r_x^2y^2 - r_x^2r_y^2$$

which has the following properties:

$$f_{\text{ellipse}}(x, y) \begin{cases} < 0, & \text{if } (x, y) \text{ is inside the ellipse boundary} \\ = 0, & \text{if } (x, y) \text{ is on the ellipse boundary} \\ > 0, & \text{if } (x, y) \text{ is outside the ellipse boundary} \end{cases}$$

we select the next pixel along the ellipse according to the sign of the ellipse function evaluated at the midpoint between the two candidate pixels.

Starting at (0, r,), we take unit steps in the x direction until we reach the boundary between region 1 and region 2-(Fig. 3-20).

$$\frac{dy}{dx} = -\frac{2r_y^2x}{2r_x^2y}$$

At the boundary between region 1 and region 2, $dy/dx = -1$ and

$$2r_y^2x = 2r_x^2y$$

Therefore, we move out of region 1 whenever

$$2r_y^2x \geq 2r_x^2y$$

$$\begin{aligned} p1_k &= f_{\text{ellipse}}\left(x_k + 1, y_k - \frac{1}{2}\right) \\ &= r_y^2(x_k + 1)^2 + r_x^2\left(y_k - \frac{1}{2}\right)^2 - r_x^2\, r_y^2 \end{aligned} \tag{3-41}$$

If $p1_k < 0$, the midpoint is inside the ellipse and the pixel on scan line y_k is closer to the ellipse boundary. Otherwise, the midposition is outside or on the ellipse boundary, and we select the pixel on scan line $y_k - 1$.

At the next sampling position ($x_{k+1} + 1 = x_k + 2$), the decision parameter for region 1 is evaluated as

$$p1_{k+1} = f_{\text{ellipse}}\left(x_{k+1} + 1,\, y_{k+1} - \frac{1}{2}\right)$$

$$= r_y^2[(x_k + 1) + 1]^2 + r_x^2\left(y_{k+1} - \frac{1}{2}\right)^2 - r_x^2 r_y^2$$

or

$$p1_{k+1} = p1_k + 2r_y^2(x_k + 1) + r_y^2 + r_x^2\left[\left(y_{k+1} - \frac{1}{2}\right)^2 - \left(y_k - \frac{1}{2}\right)^2\right] \qquad (3\text{-}42)$$

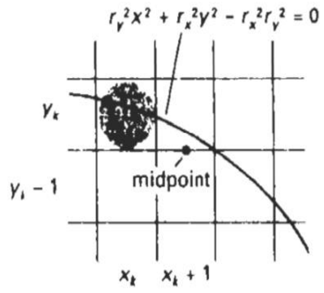


Figure 3-21
Midpoint between candidate pixels at sampling position x_k+1 along an elliptical path.

where y_{k+1} is either y_k or $y_k - 1$, depending on the sign of $p1_k$.

Decision parameters are incremented by the following amounts:

$$\text{increment} = \begin{cases} 2r_y^2 x_{k+1} + r_y^2, & \text{if } p1_k < 0 \\ 2r_y^2 x_{k+1} + r_y^2 - 2r_x^2 y_{k+1}, & \text{if } p1_k \geq 0 \end{cases}$$

As in the circle algorithm, increments for the decision parameters can be calculated using only addition and subtraction, since values for the terms $2r_y^2x$ and $2r_x^2y$ can also be obtained incrementally. At the initial position $(0, r_y)$, the two terms evaluate to

$$2r_y^2x = 0 \qquad (3\text{-}43)$$

$$2r_x^2y = 2r_x^2r_y \qquad (3\text{-}44)$$

As x and y are incremented, updated values are obtained by adding $2r_y^2$ to 3-43 and subtracting $2r_x^2$ from 3-44. The updated values are compared at each step, and we move from region 1 to region 2 when condition 3-40 is satisfied.

In region 1, the initial value of the decision parameter is obtained by evaluating the ellipse function at the start position $(x_0, y_0) = (0, r_y)$:

$$p1_0 = f_{\text{ellipse}}\left(1, r_y - \frac{1}{2}\right)$$

$$= r_y^2 + r_x^2\left(r_y - \frac{1}{2}\right)^2 - r_x^2r_y^2$$

$$p1_0 = r_y^2 - r_x^2 r_y + \frac{1}{4} r_x^2 \tag{3-45}$$

Over region 2, we sample at unit steps in the negative y direction, and the midpoint is now taken between horizontal pixels at each step (Fig. 3-22). For this region, the decision parameter is evaluated as

$$\begin{aligned} p2_k &= f_{\text{ellipse}}\left(x_k + \frac{1}{2}, y_k - 1\right) \\ &= r_y^2\left(x_k + \frac{1}{2}\right)^2 + r_x^2(y_k - 1)^2 - r_x^2 r_y^2 \end{aligned} \tag{3-46}$$

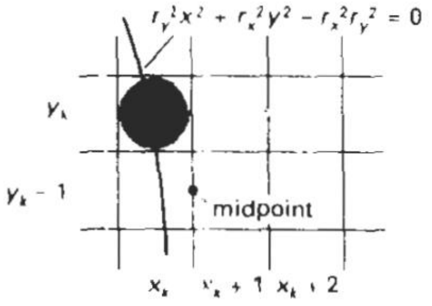


Figure 3-22
Midpoint between candidate pixels at sampling position $y_k - 1$ along an elliptical path.

If $p2_k > 0$, the midposition is outside the ellipse boundary, and we select the pixel at x_k. If $p2_k \le 0$, the midpoint is inside or on the ellipse boundary, and we select pixel position x_{k+1}.

To determine the relationship between successive decision parameters in region 2, we evaluate the ellipse function at the next sampling step $y_{k+1} - 1 = y_k - 2$:

$$p2_{k+1} = f_{\text{ellipse}}\left(x_{k+1} + \frac{1}{2}, y_{k+1} - 1\right)$$

$$= r_y^2\left(x_{k+1} + \frac{1}{2}\right)^2 + r_x^2[(y_k - 1) - 1]^2 - r_x^2 r_y^2 \qquad (3\text{-}47)$$

or

$$p2_{k+1} = p2_k - 2r_x^2(y_k - 1) + r_x^2 + r_y^2\left[\left(x_{k+1} + \frac{1}{2}\right)^2 - \left(x_k + \frac{1}{2}\right)^2\right] \qquad (3\text{-}48)$$

with x_{k+1} set either to x_k or to $x_k + 1$, depending on the sign of $p2_k$.

When we enter region 2, the initial position (x_0, y_0) is taken as the last position selected in region 1 and the initial decision parameter in region 2 is then

$$\begin{aligned} p2_0 &= f_{\text{ellipse}}\left(x_0 + \frac{1}{2}, y_0 - 1\right) \\ &= r_y^2\left(x_0 + \frac{1}{2}\right)^2 + r_x^2(y_0 - 1)^2 - r_x^2 r_y^2 \end{aligned} \qquad (3\text{-}49)$$

Midpoint Ellipse Algorithm

1. Input r_x, r_y, and ellipse center (x_c, y_c), and obtain the first point on an ellipse centered on the origin as

$$(x_0, y_0) = (0, r_y)$$

2. Calculate the initial value of the decision parameter in region 1 as

$$p1_0 = r_y^2 - r_x^2 r_y + \frac{1}{4} r_x^2$$

3. At each x_k position in region 1, starting at $k = 0$, perform the following test: If $p1_k < 0$, the next point along the ellipse centered on (0, 0) is (x_{k+1}, y_k) and

$$p1_{k+1} = p1_k + 2r_y^2 x_{k+1} + r_y^2$$

Otherwise, the next point along the circle is $(x_k + 1, y_k - 1)$ and

$$p1_{k+1} = p1_k + 2r_y^2 x_{k+1} - 2r_x^2 y_{k+1} + r_y^2$$

with

$$2r_y^2 x_{k+1} = 2r_y^2 x_k + 2r_y^2, \qquad 2r_x^2 y_{k+1} = 2r_x^2 y_k - 2r_x^2$$

and continue until $2r_y^2 x \geq 2r_x^2 y$.

4. Calculate the initial value of the decision parameter in region 2 using the last point (x_0, y_0) calculated in region 1 as

$$p2_0 = r_y^2\left(x_0 + \frac{1}{2}\right)^2 + r_x^2(y_0 - 1)^2 - r_x^2 r_y^2$$

5. At each y_k position in region 2, starting at $k = 0$, perform the following test: If $p2_k > 0$, the next point along the ellipse centered on (0, 0) is $(x_k, y_k - 1)$ and

$$p2_{k+1} = p2_k - 2r_x^2 y_{k+1} + r_x^2$$

Otherwise, the next point along the circle is $(x_k + 1, y_k - 1)$ and

$$p2_{k+1} = p2_k + 2r_y^2 x_{k+1} - 2r_x^2 y_{k+1} + r_x^2$$

using the same incremental calculations for x and y as in region 1.

6. Determine symmetry points in the other three quadrants.
7. Move each calculated pixel position (x, y) onto the elliptical path centered on (x_c, y_c) and plot the coordinate values:

$$x = x + x_c, \qquad y = y + y_c$$

8. Repeat the steps for region 1 until $2r_y^2 x \geq 2r_x^2 y$.

Example 3-3 Midpoint Ellipse Drawing

Given input ellipse parameters $r_x = 8$ and $r_y = 6$, we illustrate the steps in the midpoint ellipse algorithm by determining raster positions along the ellipse path in the first quadrant. Initial values and increments for the decision parameter calculations are

$$2r_y^2 x = 0 \qquad \text{(with increment } 2r_y^2 = 72)$$

$$2r_x^2 y = 2r_x^2 r_y \qquad \text{(with increment } -2r_x^2 = -128)$$

For region 1: The initial point for the ellipse centered on the origin is $(x_0, y_0) = (0, 6)$, and the initial decision parameter value is

$$p1_0 = r_y^2 - r_x^2 r_y + \frac{1}{4} r_x^2 = -332$$

Successive decision parameter values and positions along the ellipse path are calculated using the midpoint method as

k	$p1_k$	(x_{k+1}, y_{k+1})	$2r_y^2x_{k+1}$	$2r_x^2y_{k+1}$
0	−332	(1, 6)	72	768
1	−224	(2, 6)	144	768
2	−44	(3, 6)	216	768
3	208	(4, 5)	288	640
4	−108	(5, 5)	360	640
5	288	(6, 4)	432	512
6	244	(7, 3)	504	384

We now move out of region 1, since $2r_y^2x > 2r_x^2y$.

For region 2, the initial point is $(x_0, y_0) = (7, 3)$ and the initial decision parameter is

$$p2_0 = f\left(7 + \frac{1}{2}, 2\right) = -151$$

The remaining positions along the ellipse path in the first quadrant are then calculated as

k	$p2_k$	(x_{k+1}, y_{k+1})	$2r_y^2x_{k+1}$	$2r_x^2y_{k+1}$
0	−151	(8, 2)	576	256
1	233	(8, 1)	576	128
2	745	(8, 0)	—	—

A plot of the selected positions around the ellipse boundary within the first quadrant is shown in Fig. 3-23.

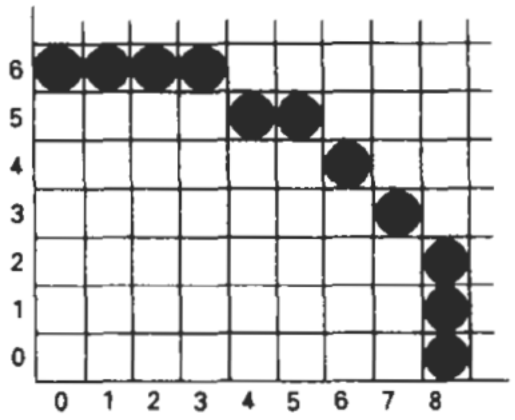


Figure 3-23
Positions along an elliptical path centered on the origin with $r_x = 8$ and $r_y = 6$ using the midpoint algorithm to calculate pixel addresses in the first quadrant.